UNIVERSITY OF

MARYLAND

Lecture Series No. 34

SOME ASPECTS OF TRANSITION FROM LAMINAR TO TURBULENT FLOW

given by

Hugh L. Dryden
Director, National Advisory Committee for Aeronautics
(Lecture at University of Maryland)

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THE INSTITUTE FOR FLUID DYNAMICS *and* APPLIED MATHEMATICS

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Introduction

It is well known that the flow of real fluids may in some circumstances approximate the flow described by the theoretical equations for a non-viscous incompressible fluid. In other circumstances the steady flow solution of the Navier-Stokes equations for a viscous imcompressible fluid provides a reasonably accurate description of the real flow. But often and perhaps more frequently the real flow exhibits random velocity fluctuations about the mean values which are a distinguishing feature of what we call turbulent motion. When the flow varies with time but the randomness is absent we regard the flow as non-turbulent. A further characteristic of turbulent motion is the more rapid diffusion of momentum, heat, and matter as compared with the molecular diffusion present in laminar flow.

We may recognize laminar and turbulent flow in different parts of one and the same flow field. In such cases we usually find laminar flow over the upstream parts of the fluid boundaries, including the surfaces of objects immersed in the flow, with a region of transition to turbulent flow downstream. As the flow speed is increased the transition region moves forward. We may observe the speeds at which transition of the flow occurs for various fixed probe positions as the speed is increased, or the positions at which transition occurs as the probe is moved downstream at various fixed speeds.

At high Reynolds numbers the effects of viscosity are confined to a relatively thin layer in the neighborhood of the boundary, the so-called boundary layer. The flow in this layer may be laminar or turbulent as noted by Prandtl more than 40 years ago in the early development of his boundary layer theory[1]. Since then comprehensive theoretical and experimental studies have been made of the flow in the boundary layer and of the transition from laminar to turbulent flow.

In this paper we shall discuss some aspects of boundary layer transition. We shall consider for the most part the simplest possible case, transition in the boundary layer of a smooth thin flat plate in a stream of uniform velocity and static pressure, the plate being parallel to the flow with its leading edge normal to the flow.

When transition occurs, the velocity near the plate, which at a suitable fixed distance from the plate is slowly falling with increasing distance from the leading edge, begins to increase rapidly with further increase in distance downstream. The location of the minimum velocity is commonly defined as the beginning of the transition region or more simply the transition point. The observed values of the transition Reynolds number $U_0 \ x_t/\nu$ found from the free stream velocity U_0, the distance x_t of the transition point from the leading edge, and the kinematic viscosity of the fluid vary from 90,000 to about 2,800,000 for the smooth thin flat plate without pressure gradient when immersed in various wind tunnel air streams.

The principal factor in this rather large variation in different air streams under nominally identical conditions was soon traced to variations in the residual turbulence present in the air stream. In fact a relationship has been demonstrated(2) between the transition Reynolds number Re_t and the intensity of the turbulence defined as the root mean square value of the fluctuations of the velocity of the free stream with time about its mean value. This relationship is shown in Fig. 1. It will be noted that the three components of the turbulent fluctuations have been arranged in a specified way.

The principal theoretical attack on the problem of transition studies the behavior of the flow with respect to small disturbances to determine whether the disturbances decrease or increase with time. Analyses of the variation of the energy of the disturbances with time have not led to definitive results. Calculation of the development in time of small disturbances which satisfy the Navier-Stokes equations of motion does show amplification of disturbances whose wave lengths lie within certain limits, provided the Reynolds number of the boundary layer exceeds a definite critical value dependent on the distribution of mean velocity. In the earlier experiments such amplified disturbances were not observed. Transition appeared to depend solely on the amplitude of the disturbances initially present in the free stream flow. We shall review the attempts to compute the response of the boundary layer flow to external disturbances and return later to the question of the stability of laminar flow.

Behavior of Laminar Boundary Layer in Presence of Disturbances

A first crude attempt to compute the effect of external disturbances was made by myself in 1936[3]. It was assumed that the free stream flow consisted of a constant velocity component U_o and a sinusoidal space variation along the boundary layer of amplitude 2 percent of the mean velocity, i.e.

$$\frac{U}{U_o} = 1 + 0.02 \sin\left(2\pi\frac{x}{\lambda} - \alpha\right)$$

where λ is the wave length of the disturbance and α its phase relative to the leading edge of the plate. A modified Pohlhausen method was used to compute the boundary layer flow. Computations were made for ten values of α. Fig. 2 shows the results for two values of α. Each computation referred to a steady flow independent of time, but the ensemble was considered as an approximation to a traveling wave of small wave velocity. The essential features of the results are as follows:

1. The mean speed is practically unaffected by this simplified turbulence of the external flow.
2. There are speed variations within the boundary layer which are much larger than those in the external flow. Their amplitude increases with distance from the leading edge of the plate.
3. Separation of the flow occurs at a distance from the leading edge of 3.4 to 3.9 wave lengths depending on the phase angle α. The point of separation moves back and forth within these limits during the fluctuation.
4. The separation point is a function of the amplitude of the disturbance.

The use of the Pohlhausen method came under some criticism by Prandtl because of its limitations in flows with adverse pressure gradients and in 1945[(4)] Quick and Schröder published the results of an improved computation. A method due to Schröder was used. The sinusoidal variation was assumed to begin after a certain distance L_0 from the leading edge for which the velocity was constant. The amplitude was chosen as 1/2 percent of the mean velocity and the wave length λ as 0.072 L_0. For this case separation occurred in the fourth wave and it was concluded that every undulation, even the weakest, finally leads to reversed flow if only the calculation is carried sufficiently far. The boundary layer proves to be extremely sensitive to small periodical oscillations in pressure, responding with large variations in its displacement thickness. Fig. 3 portrays the streamlines for this case.

An investigation of the influence of small waviness of the wall published by Görtler in 1947[5] gave similar results. The waviness of the wall has a very large effect on the boundary layer flow. If the flow continues over a sufficient number of wave lengths, separation of the boundary layer occurs.

These computations lend support to a view expressed by myself in 1931[6] that transition is due to the occurrence of intermittent flow separation. Separation yields free shear layers in the fluid which are unstable and roll-up into small-scale vortices which spread throughout the fluid. Taylor in 1938[7] derived a formula for the functional relation between the transition Reynolds number and the intensity and scale of the free stream turbulence based on the assumption that transition resulted from momentary separation produced by the pressure gradients associated with the turbulence. This relation satisfactorily represents the available experimental data for turbulence levels of the free stream exceeding 0.2 percent of the mean velocity.

There is ample opportunity for some competent mathematician to make more satisfying computations. The boundary layer equations should be solved for more realistic boundary conditions, for example for the case in which

$$\frac{U}{U_o} = 1 + \alpha \sin \frac{2\pi}{\lambda} (x-ct)$$

utilizing the equations for unsteady flow. In actual flows the external flow consists of a mean flow on which is superposed isotropic turbulence involving a stochastic process. The amplitude of the superposed turbulence could be assumed very small.

Stability of Laminar Flow

We return now to the small disturbance theory of the stability of laminar boundary layer flow. The predicted amplified disturbances were first observed by Schubauer and Skramstad in 1940[8] in an air stream for which the turbulence level was only a few hundredths of one percent of the mean speed.

It is assumed that a two-dimensional boundary layer is subjected to a two-dimensional disturbance in the form of a traveling wave moving in the direction of the mean flow. The disturbance is assumed two-dimensional in view of Squire's statement[9] that a three-dimensional disturbance* of a two-dimensional flow becomes unstable at a higher Reynolds number than the two-dimensional disturbance. The stream function of the disturbance may be written

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* Actually Squire's so-called three-dimensional disturbance was a two-dimensional wave traveling at an angle to the stream direction, i.e. the three components of the disturbance velocity were periodic in the time coordinate and in the two space coordinates parallel to the plate.

in the form $\psi(x, y) = \phi(y) e^{i\alpha(x-ct)}$ where α is real but c is complex and equal to $c_r + ic_i$. c_r is the phase velocity, α is 2π divided by the wave length, and c_i is the amplification or damping factor depending on its sign. Assuming ψ to be small, introducing non-dimensional variables, and substituting ψ in the Navier-Stokes equation, there results the so-called disturbance differential equation:

$$(U - c)(\phi'' - \alpha^2 \phi) - U'' \phi = - \frac{i}{\alpha Re} (\phi'''' - 2\alpha^2 \phi'' + \alpha^4 \phi)$$

where Re is the Reynolds number. The boundary conditions are

$$\text{at } y = o : u^1 = v^1 = o : \phi = o, \phi^1 = o$$

$$\text{at } y = \infty : u^1 = v^1 = o : \phi = o, \phi^1 = o$$

The stability problem is thus reduced to an eigen-value problem. Given the mean flow U, the Reynolds number Re, and the wave length $\lambda = \frac{2\pi}{\alpha}$ the equation yields a solution only for certain values of c_r and c_i. The possible values can be plotted on two graphs for α and c_r as functions of Re for various values of c_i. The curves for $c_i = o$ are the neutral curves separating regions of damping from regions of amplification. The results may also be expressed in terms of the frequency which equals the product of wave length by phase velocity. Fig. 4 shows the results compared with measurements by Schubauer and Skramstad[8] of the behavior of artificially excited waves.

I shall not attempt to review the mathematical aspects of the solution of the disturbance equation. Schlichting's book on Boundary Layer Theory outlines the methods used and gives references to more detailed studies of the mathematical aspects by Tollmien[10] and by Holstein[11].

When the existing disturbance is a composite one, the selective amplification isolates a wave containing a narrow band of frequencies in the neighborhood of the frequency most highly amplified. This has been observed in natural transition.

The initial disturbance spectrum consists of free stream turbulence, sound waves, and disturbances from surface roughness. For turbulence and distributed surface roughness the spectrum is constant only in a statistical sense. We have a stochastic process with a certain degree of randomness. Sound waves give noticeable effects only when the turbulence level is less than a few hundredths of one percent and the surface roughness is much less than the boundary layer thickness, say less than 0.1 the displacement thickness.

The laminar boundary layer is unstable only in the sense that the initial disturbances within a certain frequency band grow with increasing distance downstream. If the initial disturbance is reduced, the boundary layer appears more stable. But since the boundary layer is so extremely sensitive to external disturbances, we probably cannot avoid instability at sufficiently high Reynolds numbers, at least for the case of incompressible flow without heat transfer. The case of compressible flow with cooling is discussed later.

Origin of Turbulence

The amplified disturbances do not constitute turbulence and the breakdown of laminar flow does not constitute transition to turbulence. There are many examples of the breakdown of laminar flow into a flow varying periodically with the time, often with regular vortex patterns. Examples which have been treated theoretically and observed experimentally are (1) the Kármán vortex street behind a cylinder; (2) the Taylor three-dimensional vortex cells between two concentric rotating cylinders; and (3) the Görtler vortices near a concave surface. These periodic patterns are well defined and essentially laminar in character at Reynolds numbers which do not exceed too greatly that for instability of the steady laminar flow.

Turbulence is characterized by random velocity fluctuations at a point and by greatly increased diffusion of momentum, matter and heat. Transition has not occurred until turbulence as thus defined is present.

The riddle of the origin of turbulence is one which has engaged the attention of many throughout past decades. The fact that vorticity is generated through the action of viscosity in the region near solid boundaries, i.e. in boundary layers is now fairly clear. St. Venant was the first to note that vorticity cannot be generated in the interior of a viscous incompressible fluid, subject to conservative extraneous forces, but is necessarily diffused inward from the boundaries(12). Truesdell(13) has given us a comprehensive treatment of the kinematics of vorticity.

Betz[14] has described some of the difficulties in devising a conceptual model of the formation of discrete vortices. The time required to produce a given circulation is inversely proportional to the viscosity and directly proportional to the square of the diameter of the vortex core. The time is very short for vortex cores of small diameter. However the energy of the flow increases without limit as the diameter approaches zero. Betz concludes that discrete vortices are formed by the rolling up of thin vortex sheets. Such sheets can be produced in flows with very small viscosity. When a boundary layer separates from the surface in a region of adverse pressure gradient it becomes a thin vortex layer in the free fluid.

Helmholtz[15] and Rayleigh[16] discussed the instability of surfaces of discontinuity in an incompressible frictionless fluid and showed that any small disturbance increases in amplitude with time. Rosenhead[17] attempted to follow the motion in its later stages and showed the rolling up of the layer and concentration of the vorticity at discrete points. The wave length which finally dominates, i.e. the vortex spacing, can only be determined by a stability computation which includes the effects of viscosity.

Experimental attack on the problem of the origin of turbulence is yielding suggestive results which should aid in the development of an adequate theory. It has been known since 1936[3] that transition to turbulence in a boundary layer occurs suddenly just as Reynolds observed in pipe flow with a filament of dye. The turbulent bursts occur randomly and infrequently near the upstream limit of their occurrence and become more frequent and of longer duration further downstream until finally the flow is always turbulent. These observations were interpreted as a to and fro motion of the transition point.

In 1951[18] Emmons proposed, as a result of observations on a thin sheet of water flowing down an inclined plate, that turbulence originated in spots which grew in size as they moved downstream. Using a technique developed by Mitchner[19], Schubauer and Klebanoff[20] have studied the behavior of turbulent spots induced by a spark discharge. These show a characteristic oscillograph signature when they sweep over a hot-wire probe and the same signature appears in records of natural transition. There seems little doubt that transition to turbulence does in fact originate in many local regions which grow in size until they merge as the turbulent fluid moves downstream. The observed intermittency is due to the passage of such spots past the probe.

To study the detailed phenomena more systematically Schubauer and Klebanoff returned to the technique by which they had confirmed the stability theory. Laminar boundary layer oscillations within the amplified range of frequency were induced by a vibrating ribbon whose amplitude could be increased with time so that successive stages of the instability occurred in sequence at a fixed probe position. This work is still in progress but many interesting results have been obtained. As the amplitude of the exciting ribbon is increased, the amplitude of the nearly sinusoidal oscillations increases and the wave form becomes distorted. Unexpectedly however a three-dimensional disturbance is superposed, made apparent(Fig. 5) in a variation of the amplitude of the two-dimensional waves along their length.

These observations suggest a further theoretical study of the stability of the boundary layer for three-dimensional disturbances. Although Squire[9] stated that the two-dimensional instability appears at a lower Reynolds number, the margin may be very small. The experiments also suggest a direct experimental study utilizing a localized "point" source in place of the two-dimensional ribbon.

As the amplitude of the ribbon was further increased, secondary disturbances appeared in the crests and troughs of the two-dimensional waves(Fig. 5). At the transverse location of minimum amplitude regular high frequency ripples appeared on the somewhat flattened wave crests (i.e. high velocity part of the cycle) with the troughs remaining smooth(Fig. 6). At the transverse location of maximum amplitude large irregular high frequency disturbances, perhaps to be interpreted as turbulence, appeared in the troughs while the flattened crests showed no disturbance(Fig. 5).

The latter observation is consistent with the intermittent separation theory of the origin of turbulence. The first observations suggested the generation of three-dimensional vortices of the Görtler type in the region of concave flow curvature as proposed by Görtler[21]. It is hoped that further work will clarify the physical interpretation of these secondary phenomena. A theoretical study of the stability for a boundary layer with a velocity distribution corresponding to the amplified Tollmien-Schlichting waves would be very helpful.

In an actual flow the peak amplitudes would probably not be so uniformly distributed as found for the carefully controlled experiments with artificially excited disturbances. Therefore the turbulent spots would originate at points randomly distributed. The pattern would probably resemble that of the white caps in the open ocean where we are observing a complex wave system with peak amplitudes randomly distributed. However the key process in the origin of turbulence appears to be associated with a rather localized three-dimensional disturbance of the boundary layer and theoretical attack should be concentrated on this process.

Spread of Turbulence

Schubauer and Klebanoff[20] have studied the growth of artificially generated turbulent spots. The spots were produced by a spark discharge through the boundary layer at a point where the boundary layer Reynolds number was below the transition Reynolds number but above the minimum critical Reynolds number. Specifically the minimum Reynolds number was 450, the natural transition Reynolds number about 2900, and the Reynolds number at the spot probe was about 2100, all based on the displacement thickness. The shape of the growing spot, shown in Fig. 7, is approximately conical in plan and thus characteristic of a moving source which is propagating a flow disturbance in all directions at a speed less than the speed of the source. This speed of propagation of transition at the boundary may be computed from the speeds at the leading and trailing edges of the spot, from the vertex angle, and from the angle subtended at the origin. The three values are in approximate agreement ranging from 0.19 to 0.22 times the mean flow velocity. This speed is considerably less than the speed of the most highly amplified Tollmien-Schlichting waves which lies within the limits 0.27 to 0.35 times the mean flow velocity at a Reynolds number of 2100.

The speed of spread of turbulence can also be computed from the angle of the turbulent wedge behind a single three-dimensional roughness element. Turbulent wedge angles observed under various conditions vary from 8.5° to 11° corresponding to speeds of 0.15 to 0.19 times the mean flow velocity.

I believe that the growth of the turbulent spot in a laminar boundary layer is the effect of the flow disturbances surrounding the turbulent spot in breaking down the adjacent laminar flow. From the results of measurements of the effect of external turbulence on transition, it is computed that a disturbance amounting to 0.35 percent of the mean speed would produce transition at a Reynolds number of 2100. The finite speed must be associated with the finite amplification rate of the disturbances. The theoretical computation of this speed of propagation of transition is a worthy task for some mathematician.

If this speculation is correct, an artificially generated spot should not grow at a location where the boundary layer Reynolds number is less than the minimum critical Reynolds number. Schubauer found that the spot grew at a slow rate until the boundary layer Reynolds number exceeded 450 and then grew at a much faster rate.

(Note: The remainder of this paper is a translation of sections of a paper, "Neuere Untersuchungen der Frage des Umschlages", presented by the author at the Annual Meeting of the Deutsche Versuchsanstalt für Luftfahrt, Munich, September 30, 1955. A brief abstract of this material was given in the oral presentation of the University of Maryland lecture.)

Theory of the Stability of Compressible Boundary Layer Flow

In 1946 and 1947 Lees and Lin[22, 23] extended the Tollmien-Schlichting theory to a compressible laminar boundary layer. The effect of increasing Mach number on the stability limit on a flat plate with no heat transfer is a slow decrease in the critical Reynolds number. Heating the body above the recovery temperature (that attained by the plate without heat transfer under equilibrium conditions) reduces the critical Reynolds number, whereas cooling the body decreases it. A most important conclusion was that the boundary layer could be completely stabilized by sufficient cooling of the body. These computations have given rise to much controversy as to their accuracy and the validity of the assumptions.

Dunn and Lin[24] have recently published a more complete theory, removing some of the earlier limitations and modifying some of the general conclusions of the earlier treatment. They find that at a Mach number between 1 and 2 three-dimensional disturbances begin to play the leading role in producing transition. At supersonic Mach numbers the boundary layer can never be completely stabilized by cooling with respect to all three-dimensional disturbances. However in many cases surface cooling is still a very effective means of stabilizing the boundary layer. According to their computations the critical wall to free stream temperature ratio required for complete stability with respect to two-dimensional disturbances is 1.6 at a Mach number of 3; for a Mach number of 4 the critical temperature ratio is 1.7 and the ratio falls to zero at a Mach number of 7.5. The critical temperature ratio for complete stability against three-dimensional disturbances corresponding to oblique waves varies with the direction of propagation. At a Mach number of 4 the values for angles of 0° to 74° to the flow direction vary between 1.47 and 1.7 and fall to zero at angles of 75° or more. There is still some uncertainty in these results since the computations have been restricted to small wave numbers.

Experimental Studies of Stabilization by Cooling

Within the last five years there have been several attempts to demonstrate experimentally the complete stabilization by cooling which was predicted by the earlier theoretical work, and research on this question is very active at the NACA and elsewhere in the U.S.

The experimental data available show clearly that heat transfer from the body to the airstream decreases the transition Reynolds number whereas heat flow from the airstream to the body increases the transition Reynolds number. The magnitude of the effects increases with increase in transition Reynolds number of the insulated body, i.e. with decrease in air stream turbulence and surface roughness and with body shapes giving favorable pressure distribution.

Fig. 8 shows a sampling of the experimental data plotted in terms of the observed transition Reynolds number as function of the ratio of the difference between the wall temperature T_w and the recovery temperature T_r to the stagnation temperature T_o. The data in the literature which have been omitted from Fig. 8 relate mainly to the effects of heating or to experiments on artificially roughened bodies. The figure includes data taken at Mach numbers 1.61, 2.40, 2.87, and 3.12 on the RM-10 body of revolution, flat plate, cone-cylinder and paraboloid-cylinder, respectively.

Czarnecki and Sinclair[25] made measurements on a parabolic body of revolution (RM-10) at a Mach number of 1.61 in the NACA

Langley 4-foot wind tunnel. These experiments gave the highest transition Reynolds number for the insulated body for any of the available measurements, the Reynolds number for transition at the base being 11.5 million. These experiments show the largest effects of heating and cooling. Heating to a value of $\frac{T_w - T_r}{T_o}$ of 0.3 reduced the transition Reynolds number from 11.5 to 3 million; cooling to a $\frac{T_w - T_r}{T_o}$ of -0.15 increased it from 11.5 to 28.5 million. Roughening the surface greatly reduced the sensitivity to heat transfer effects. Cellophane tape on the body at 3 and 25 percent of the body length reduced the value for the insulated body from 11.5 to 5 and 7.5 million respectively, and these values could not be increased by cooling the roughened body.

Higgins and Pappas[26] measured the effects of heating a flat plate at a Mach number of 2.40 in the NACA Ames 6-inch heat transfer tunnel in which transition occurred on the insulated plate at a Reynolds number of 1.25 million. Heating to a wall temperature 200°F above the recovery temperature reduced the transition Reynolds number to 600,000. Eber[27] measured transition on a cone-cylinder model, the cone having a total vertex angle of 40°, in one of the NOL wind tunnels at a Mach number of 2.87. Heating to a wall temperature 125°F greater than the recovery temperature decreased the transition Reynolds number from 300,000 to 160,000 whereas cooling by 50°F increased it to 450,000.

Jack and Diaconis[28] made measurements on two bodies of revolution at a Mach number of 3.12 in the NACA Lewis one-foot wind tunnel in which the stagnation pressure could be varied from 8 to 52 lbs/in^2 and the stagnation temperature from 50° to 170°F. The ratio of wall temperature to free stream temperature could be varied from 0.7 to 4.4, the value for no heat transfer being approximately 2.7. For the cone-cylinder, heating to a value of $\frac{T_w - T_r}{T_o}$ of 0.53 decreased the transition Reynolds number from 2 million to 0.86 million; cooling to -0.45 increased it to 10.6 million. For the paraboloid-cylinder the transition Reynolds numbers were approximately twice as great.

The basic parameter used in the stability computations is the ratio of the wall temperature T_w to the free stream static temperature T_∞. The method of plotting of Fig. 8 is more convenient for most purposes since the heat transfer direction is evident. It can be shown that the relation between $\frac{T_w - T_r}{T_o}$ and $\frac{T_w}{T_\infty}$ is given by the linear relation $\frac{T_w - T_r}{T_o} = R\frac{T_w}{T_\infty} - r(1-R) - R$ where r is the recovery factor (0.851 for the laminar boundary layer) and R is the ratio $\frac{T_o}{T_\infty}$ which for isentropic flow is simply $1/(1 + 0.2\,M^2)$. Using this relationship, the Dunn and Lin theoretical values of $\frac{T_w}{T_\infty}$ for complete stabilization against two-dimensional disturbances may be converted to values of $\frac{T_w - T_r}{T_o}$. The resulting values are plotted against Mach number in Fig. 9.

The curves of Fig. 8 for the measurements of Czarnecki and Sinclair (curve A) and of Jack and Diaconis (curves B and C) suggest the presence of vertical asymptotes at sufficiently low values of $\frac{T_w - T_r}{T_o}$. Approximate values are indicated in Fig. 9. Since the Reynolds number attainable in any experiment is always finite, it will never be possible to demonstrate complete stabilization. However, a large increase in transition Reynolds number on smooth fair bodies of revolution is obtained by reducing the wall temperature by cooling to values of the order of those predicted by the theory.

The Effect of Mach Number on Transition on Bodies Without Heat Transfer

The stability theory indicates a slow reduction of the critical Reynolds number with increasing Mach number for a body without heat transfer. The experimental situation is quite confused. Wind tunnel results generally but not always show a decrease as predicted, but data obtained by firing models through the air shows an increase of critical Reynolds number with Mach number.

The wind tunnel data are greatly affected by wind tunnel turbulence, which may vary with Mach number, stagnation pressure, compressor interconnections, etc. Measurements of transition have been made on a cone of 10° total vertex angle in numerous NACA wind tunnels[29]. The values obtained vary from 400,000 to 7 million and the variation with Mach number is in some cases a decrease and in others an increase with increasing Mach number. Turbulence measurements were not available but in the NACA Lewis one-foot tunnel the transition Reynolds number has been increased from 700,000 to 1.3 million and then to about 4 million by successive modifications to reduce the turbulence level. These values are not directly comparable with those for a flat plate, the equivalent flat-plate Reynolds numbers being equal to one-third of the x-Reynolds numbers for the cone.

Brinich[30] made measurements of transition on the outer surface of a hollow cylinder with sharp leading edge in the NACA Lewis one-foot wind tunnel at a constant Mach number of 3.12 but with the Reynolds number varied by varying the stagnation pressure. The transition Reynolds number increased from 1.5 to 4 million as the pressure was increased from 6 to 52 lbs/in^2. Brinich makes the interesting suggestion that in his experiments transition was controlled by air stream turbulence rather than by development of instability. The controlling parameter is then the Taylor turbulence parameter involving the turbulence level and the ratio of the scale of the turbulence to the boundary layer thickness. Assuming a fixed turbulence level and scale, the boundary layer thickness decreases as the density increases and hence the ratio of boundary layer thickness to the scale of the turbulence decreases. Thus the turbulence of fixed scale has less effect at higher pressures in reducing the transition Reynolds number. On this view there would be both a Mach number effect and a density effect since the thickness increases with increasing Mach number. In a wind tunnel of constant stagnation pressure, increasing Mach numbers are associated with reduced density so that the two effects are additive.

Lange, Gieseler, and Lee[31] observed a decrease in transition Reynolds number for a 5° cone from 3.4 million to 1.0 million in passing from a Mach number of 1.9 to 4.2 and from 3 million to 1.0 million for a hollow cylinder. Coles[32] found in measurements on a plate in the JPL 20-inch wind tunnel a decrease from 2.25 million to 1.10 million between Mach numbers 2 and 3.6 but a rise to 1.2 million from 3.6 to 4.5.

Because of the sensitivity of the transition position to heat transfer, measurements of the effect of Mach number must be made with great care to avoid heat transfer effects. Thus measurements on models in flight are ordinarily made in so short a time that the model does not have time to heat up to the recovery temperature. The conditions are therefore those of a cooled body with the wall temperature substantially equal to the free stream static temperature.

Since the stagnation temperature increases with Mach number, the wall temperature ratio decreases and hence the transition Reynolds number increases with Mach number.

Concluding Remarks

A review has been given of some of the current work in the United States on the transition problem with special attention given to the fundamental studies of the origin of turbulence in incompressible boundary layer flow and to the stabilization of the compressible boundary layer by cooling.

Persistent attack on the fundamentals of the transition process by theoretical and experimental workers in many countries is yielding a more complete understanding of the origin of turbulence. Although much remains to be done, one has a feeling that we are now on the right track and that progress will be more rapid.

The study of transition in boundary layers in supersonic flow is making considerable progress. The difficulty of making turbulence measurements in supersonic wind tunnels is a definite handicap to the interpretation of experimental data. The theory of the stability of compressible boundary layer flow needs much more critical examination. Basic studies are notably lacking although attempts are being made by one group in the U. S. to repeat the classic Schubauer-Skramstad experiments in supersonic flow.

The contributions of German scientists to this problem have been very great, and workers in other countries look forward with anticipation to the contributions of the DVL and other German aeronautical research institutes in the future.

Figure Legends

Fig. 1. Transition Reynolds number on a smooth flat plate as a function of intensity of the free-stream turbulence.

Fig. 2. Computed distribution of mean speed in the boundary layer of a plate for the external flow

$$U/U_0 = 1 + 0.02 \sin(2\pi x/\lambda - \alpha).$$

The contours are contours of equal values of u/U_0. Separation begins at the points indicated by arrows.

Fig. 3. Stream lines for flow in the boundary layer of a plate subjected to periodically varying air speed beginning at distance L_0 from the leading edge.

$$U/U_0 = 1 + 0.005 \sin(2\pi x/0.072\, L_0)$$

for $x > L_0$.

Fig. 4. Comparison of theoretical and experimental values of frequencies of neutral Tollmien-Schlichting waves in boundary layer of a plate.

Fig. 5. Variation of amplitude of Tollmien-Schlichting waves with z - coordinate (normal to flow direction and parallel to leading edge of plate) and nature of secondary disturbances in regions of high and low amplitude.

Fig. 6. Development of secondary oscillations prior to transition as amplitude of Tollmien-Schlichting waves is increased.

Fig. 7. Growth of artificial turbulent spot in laminar boundary layer (from Reference 20).

Above -- Oscillograms of passage of spark-induced and natural-transition turbulent spots over hot-wire anemometer. Timing dots at intervals of 1/60 sec.

Below -- Plan and elevation views of spark-induced turbulent spot about 2.4 ft. downstream from origin. δ is Pohlhausen thickness of laminar boundary layer. $U_0 = 30$ ft/sec.

Fig. 8. Effect of Heating and Cooling on Transition Reynolds Number.

T_w = wall temperature, T_r = recovery temperature,

T_o = stagnation temperature.

Fig. 9. Theoretical and experimental values of $\dfrac{T_w - T_r}{T_o}$

for stabilization of compressible laminar boundary layer by cooling.

Curve represents Dunn and Lin theoretical values of stabilization against two-dimensional disturbances.

Isolated points are estimated experimental vertical asymptotes from Fig. 8.

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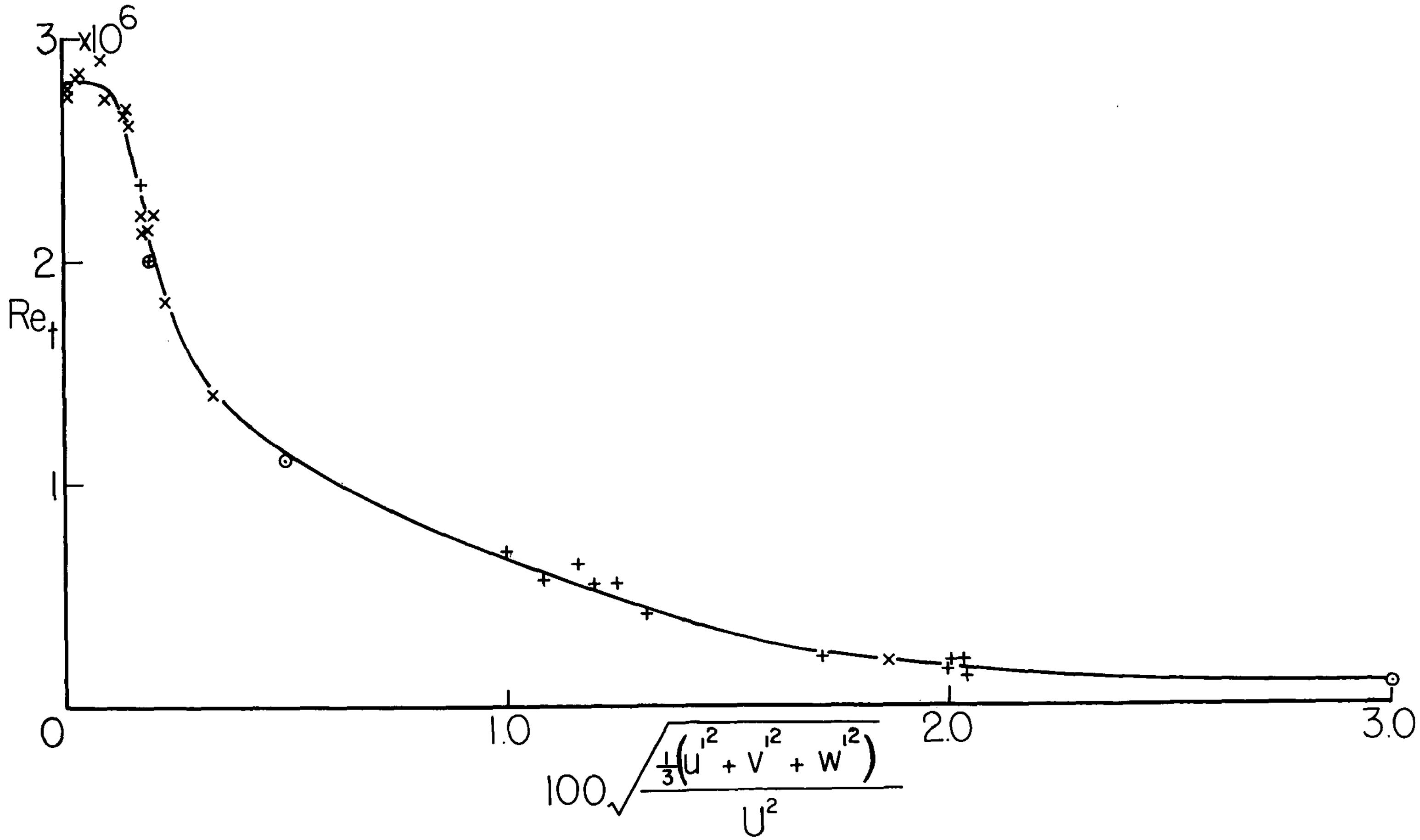

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Fig. 1. Transition Reynolds number on a smooth flat plate as a function of intensity of the free-stream turbulence.

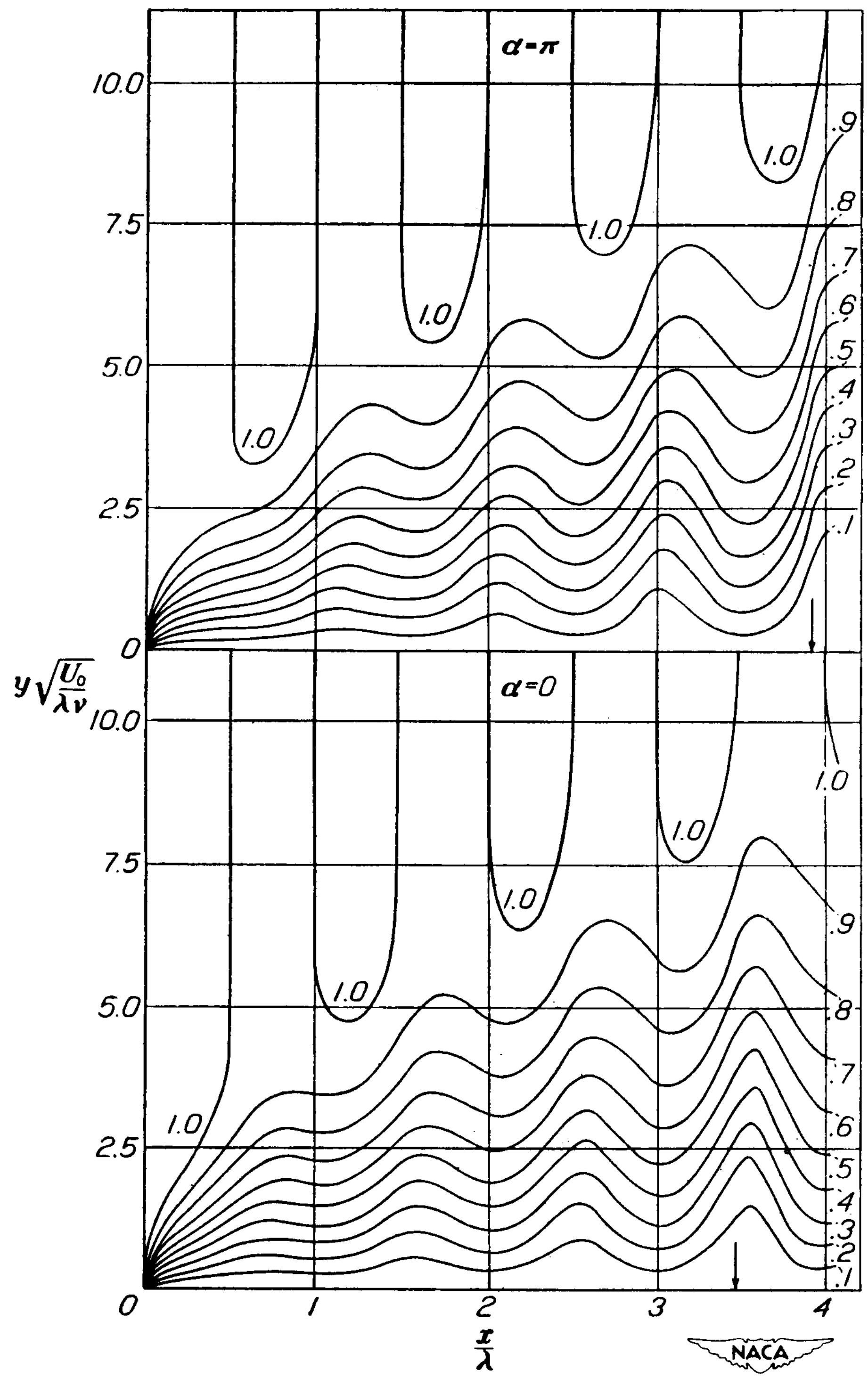

Fig. 2. Computed distribution of mean speed in the boundary layer of a plate for the external flow

$$U/U_o = 1 + 0.02 \sin (2\pi x/\lambda - \alpha).$$

The comtours are contours of equal values of u/U_o. Separation begins at the points indicated by arrows.

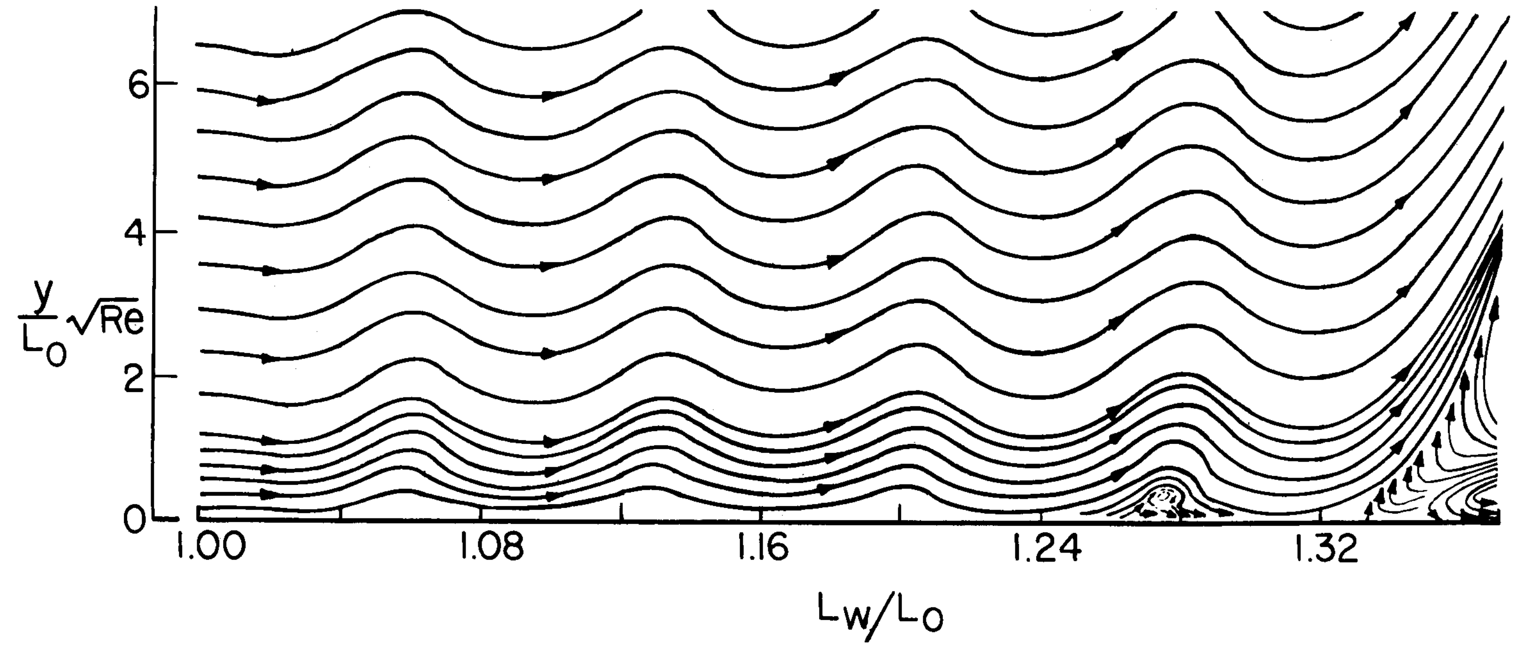

Fig. 3. Stream lines for flow in the boundary layer of a plate subjected to periodically varying air speed beginning at distance L_o from the leading edge.

$$U/U_o = 1 + 0.005 \sin (2\pi x/0.072\ L_o)$$

for $x > L_o$.

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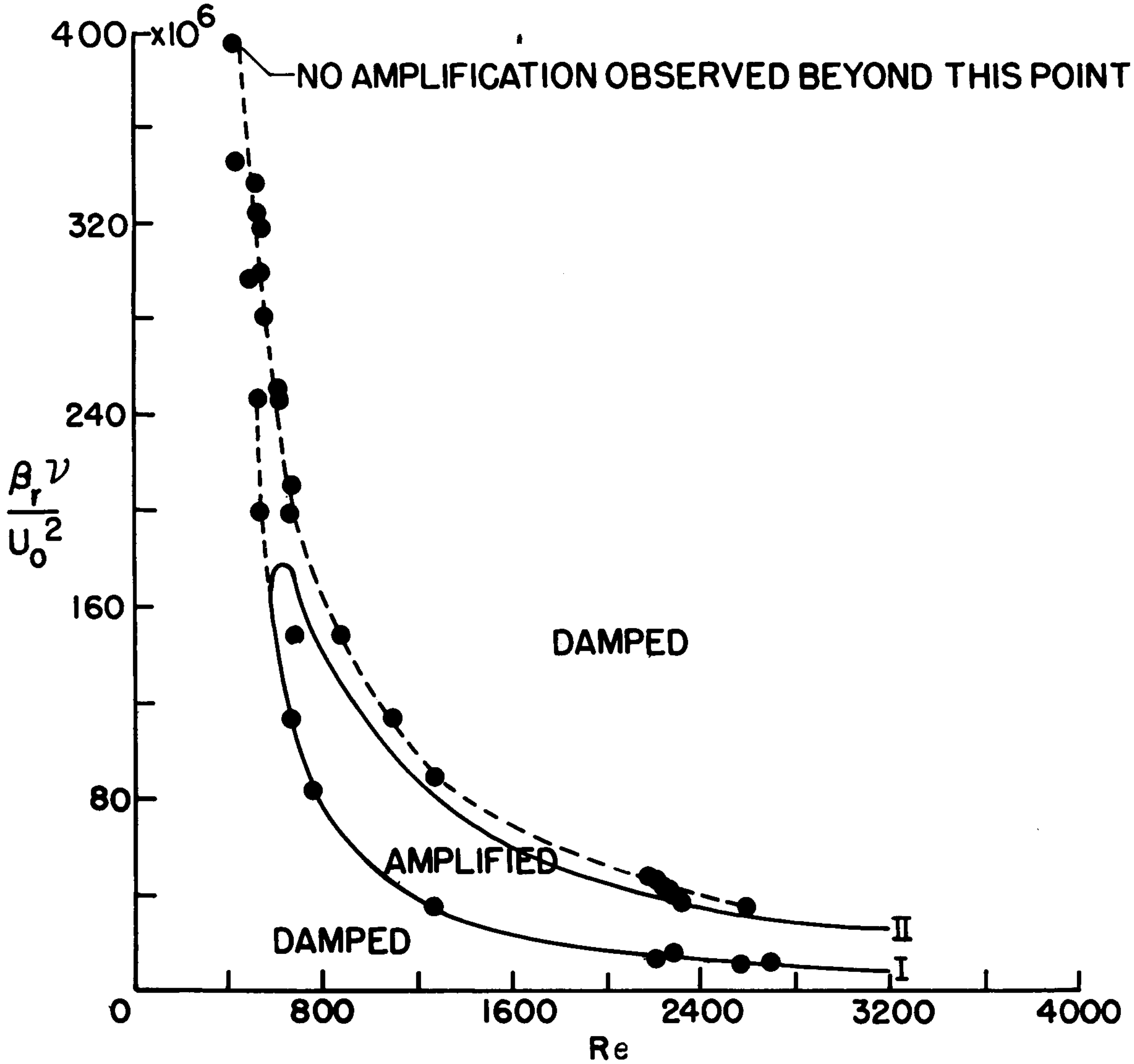

Fig. 4. Comparison of theoretical and experimental values of frequencies of neutral Tollmien-Schlichting waves in boundary layer of a plate.

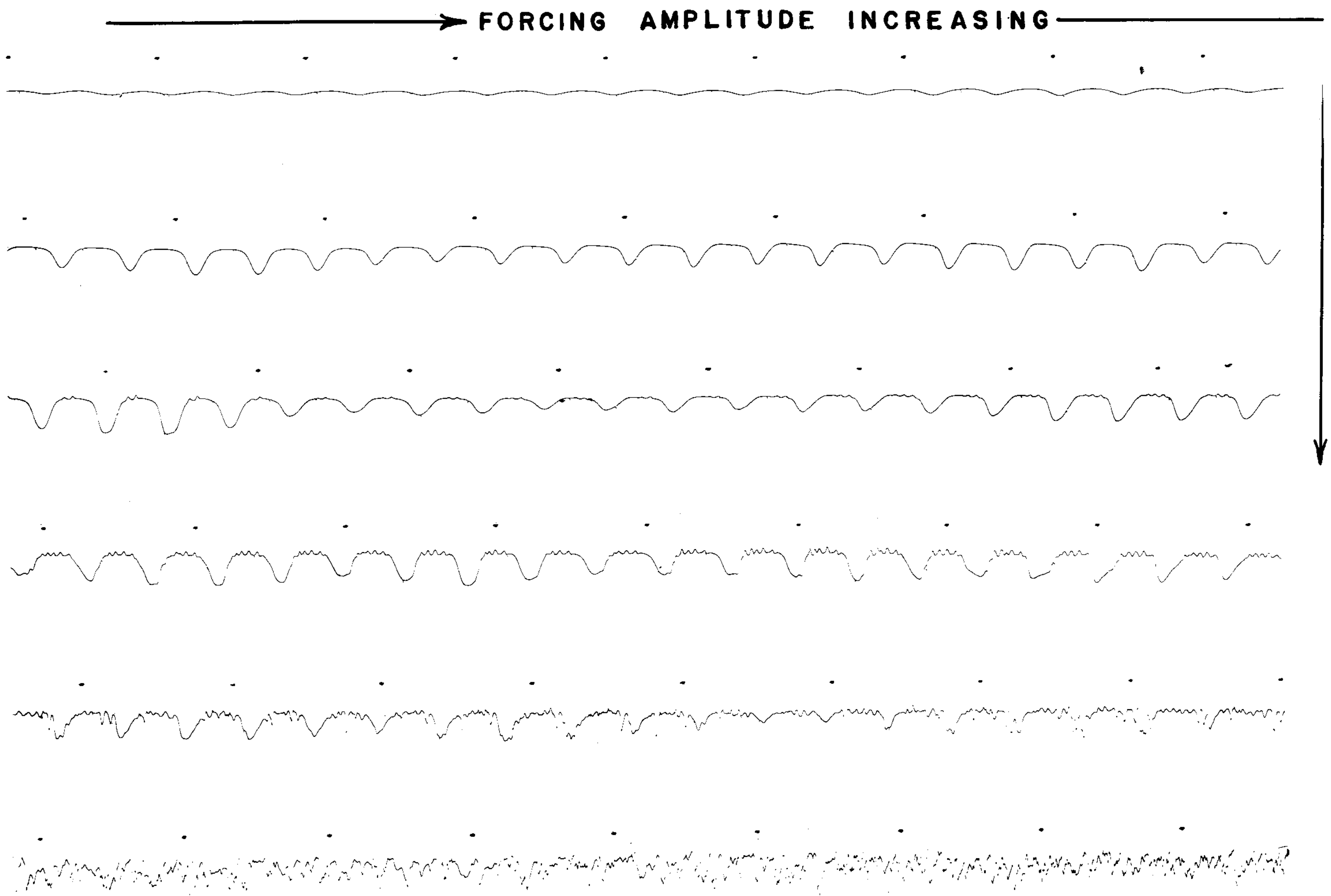

Fig. 5. Variation of amplitude of Tollmien-Schlichting waves with z-coordinate(normal to flow direction and parallel to leading edge of plate) and nature of secondary disturbances in regions of high and low amplitude.

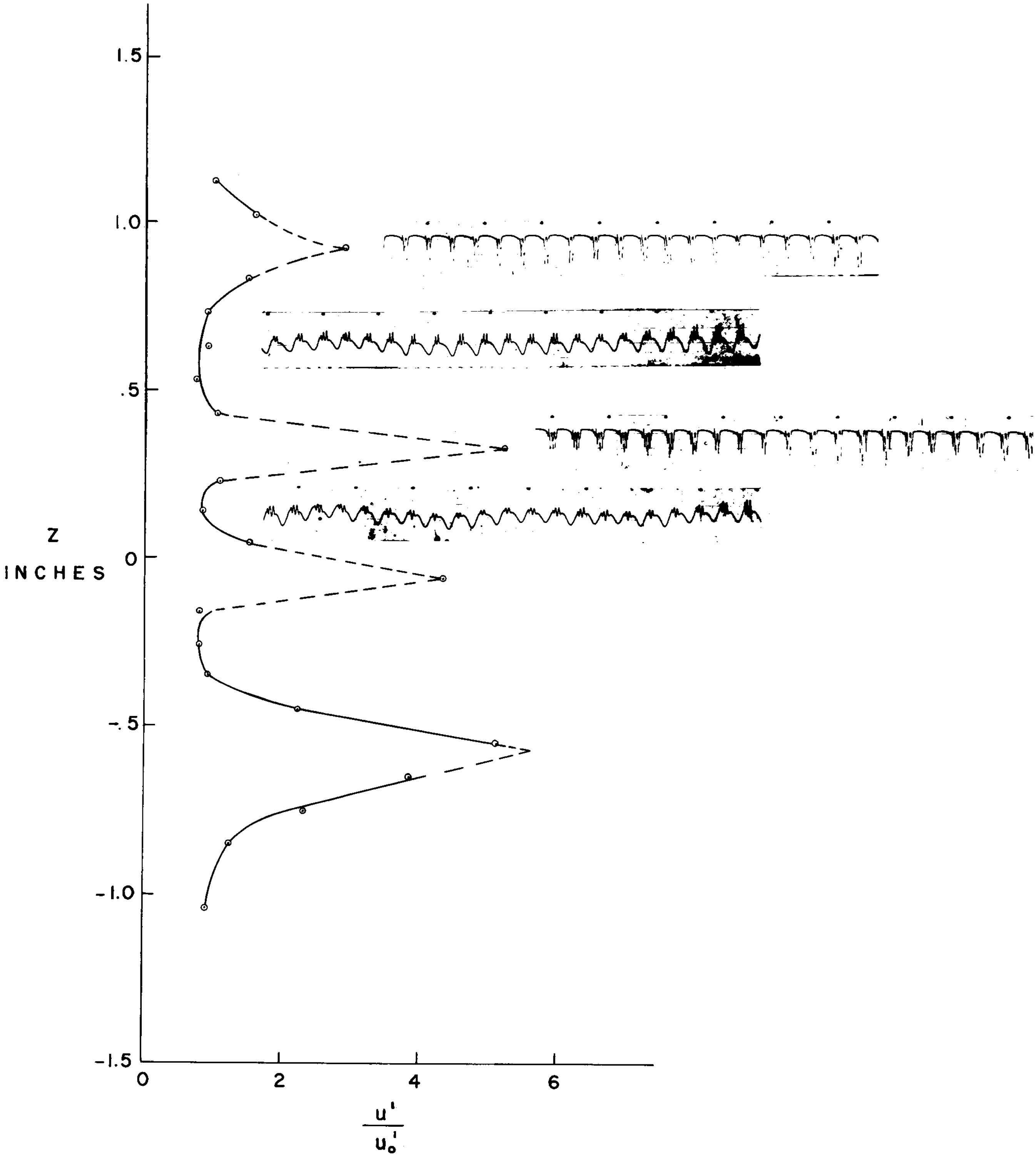

Fig. 6 Development of secondary oscillations prior to transition as amplitude of Tollmien-Schlichting waves is increased.

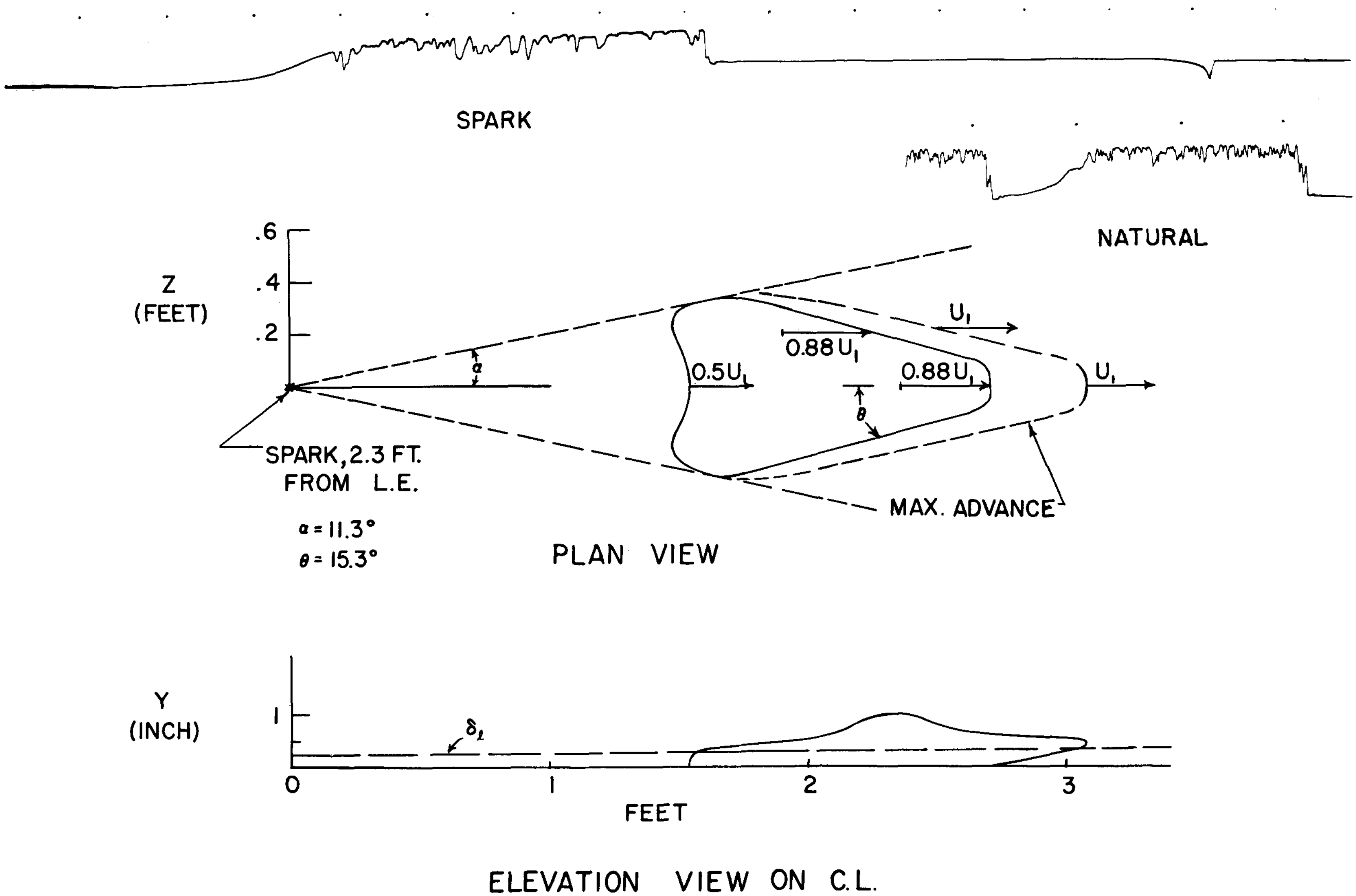

Fig. 7. Growth of artificial turbulent spot in laminar boundary layer (from Reference 20).

Above--Oscillograms of passage of spark-induced and natural-transition turbulent spots over hot-wire anemometer. Timing dots at intervals of 1/60 sec.

Below -- Plan and elevation views of spark-induced turbulent spot about 2.4 ft. downstream from origin. δ is U_0 = 30 ft/sec.

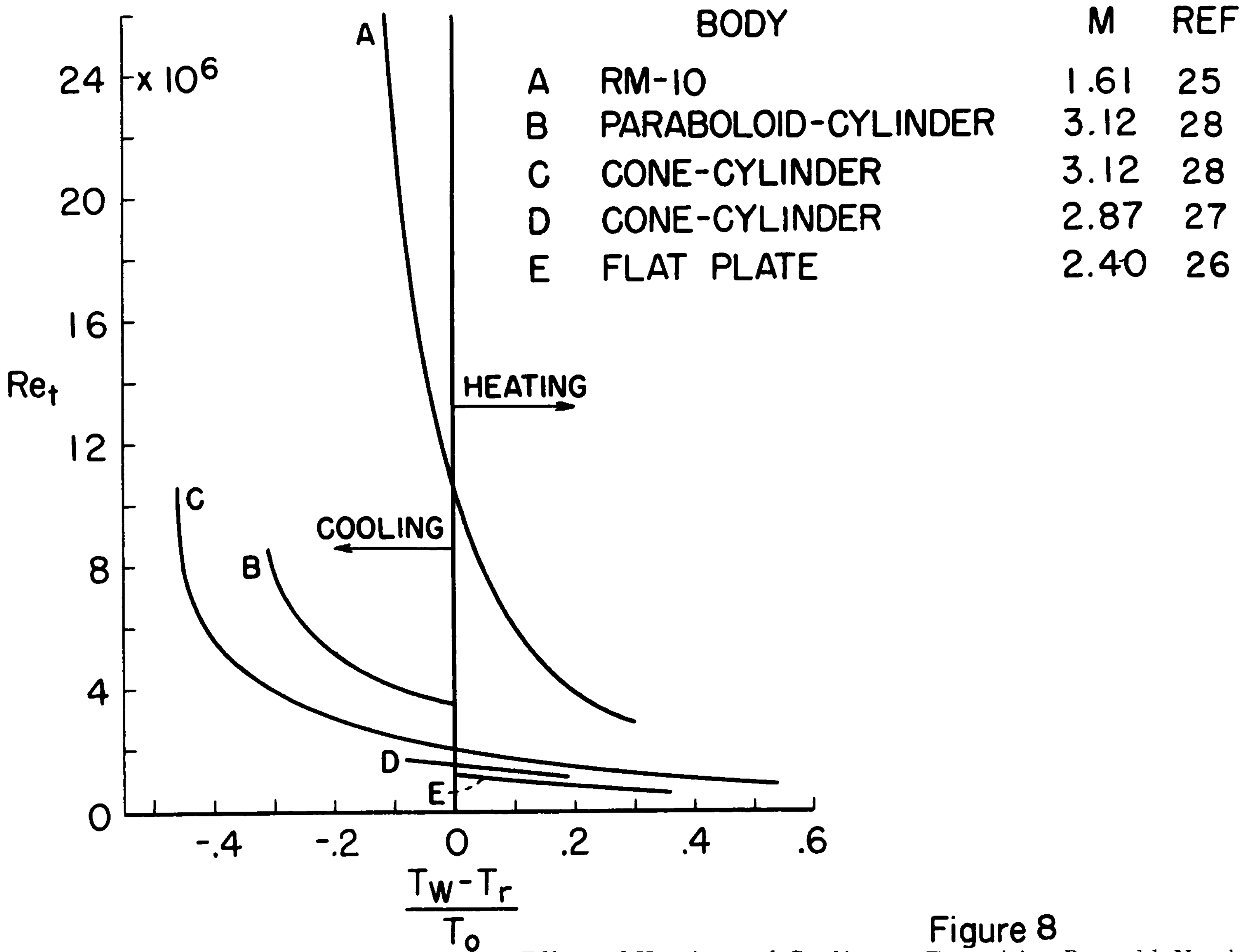

Figure 8

Effect of Heating and Cooling on Transition Reynolds Number.

T_w = wall temperature, T_r = recovery temperature,

T_o = stagnation temperature.

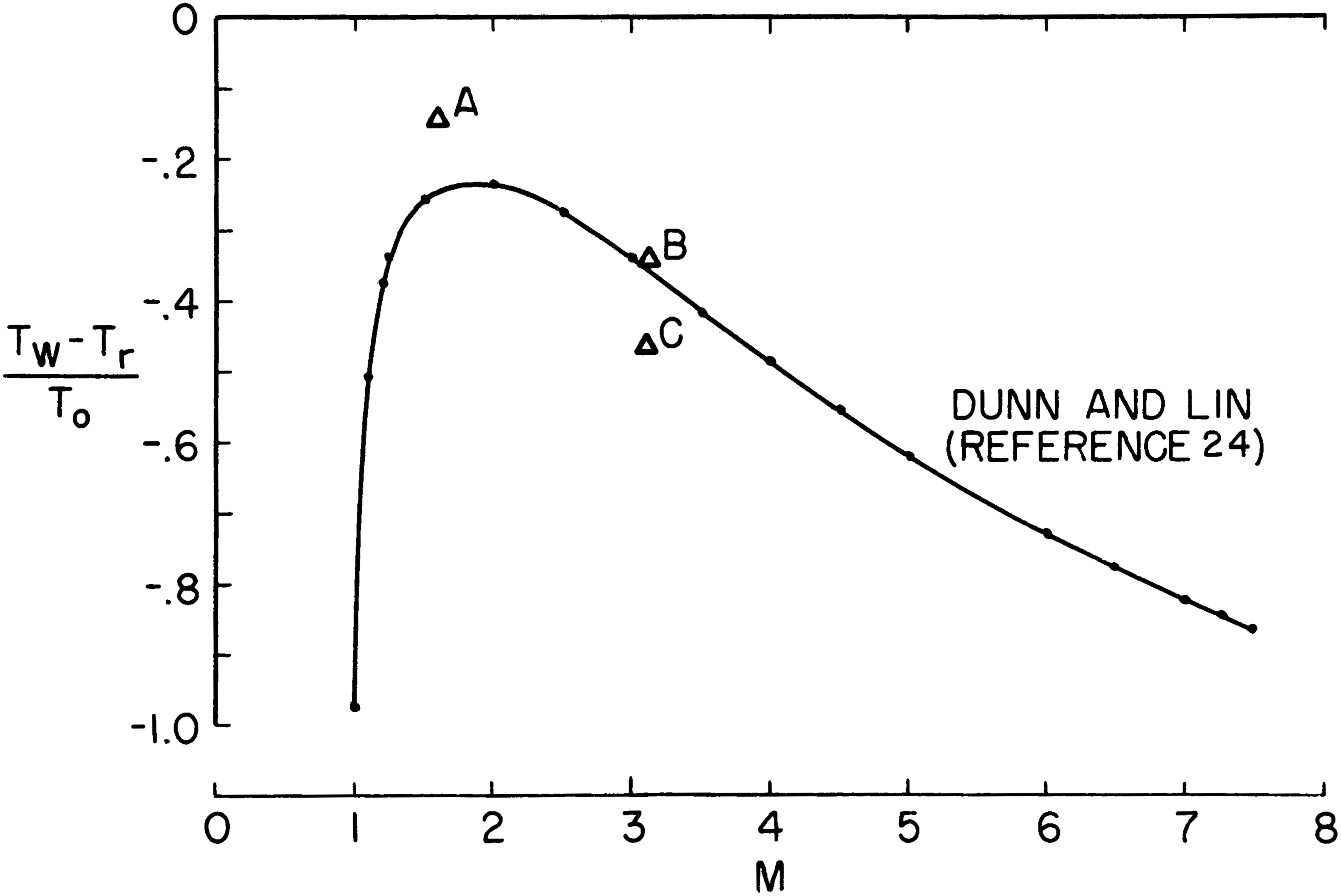

Figure 9.

Theoretical and experimental values of $\frac{T_w - T_r}{T_o}$ for stabilization of compressible laminar boundary layer by cooling. Curve represents Dunn and Lin theoretical values of stabilization against two-dimensional disturbances. Isolated points are estimated experimental vertical asymptotes from Fig. 8.